Streaming Calculations using the Point-Kernel Code RANKERN

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RANKERN solves the gamma-ray transport equation in generalised geometry using the point-kernel method, with build-up factors accounting for the contribution from scattered radiation. Developments to RANKERN have overcome the main limitation of the point-kernel method and the associated build-up data; i.e. that they are based on the calculation of the flux arising from radiation which has passed in a direct line from the source to the dose-point. This assumption is adequate for a wide range of systems that are comprised of bulk shielding. However, it is inaccurate in situations where the dominant contribution to the result arises from radiation which has undergone a number of changes in direction between source and dose-point in order to travel along weaknesses in the shield. This is the phenomenon that has to be modelled in streaming calculations.

When the direct flight from source to the detectors is at an angle to the shield, the build-up data can be altered to take account of the obliquity. Such oblique penetration build-up factors are an approximation, which work for small angles but break down when the angle of obliquity becomes large. When this occurs, intermediate points of scatter between the source and dose-point need to be included in the calculation to provide a more accurate calculation. In many cases, the mechanism of scatter in a volume can be approximated by reflection at a surface, with appropriate albedo data giving the probability of reflection.

This paper describes how RANKERN may be used to perform streaming calculations. Validation evidence is presented for the use of the code in such situations, including streaming along multi-legged ducts.

KEYWORDS: Gamma rays, point-kernel, radiation streaming, build-up, oblique penetration, scatter, reflection, albedo, validation, multi-legged ducts

I. Introduction

The design of bulk shielding is relatively straightforward. However, the penetrations through the shield can present significant problems for the radiation shield designer. As part of a collaborative programme AEA Technology and BNFL have further developed the 3D RANKERN[1] point-kernel code as a robust design tool for all gamma-ray shielding studies. The code's powerful and flexible source and geometry capabilities coupled with an efficient optimisation algorithm and novel features, such as the automatic computation of multi-layered build up factors, have led to its widespread use.

This paper describes how RANKERN deals with situations where the conventional build-up method breaks down. Specifically, these can occur when intermediate events between the source and dose-points need to be considered, as is the case in penetration along multi-legged ducts. To start the analysis we first look at the situation when penetration is oblique to the shield.

II. Oblique Penetration

RANKERN solves the point-kernel equation for any complexity of source and shield configuration. Its algorithm can be summarised by considering a source of strength S $\gamma/cm^3/s$ distributed over a volume V. The flux ϕ at a distance r following penetration through τ mfp (mean free path) of shield is given by

$$\phi = \int_V \frac{SB}{4\pi r^2} e^{-\tau} dv ,$$

where B is the build-up factor catering for scattered radiation, and subscripts relating to energy, material etc. have been omitted for clarity. In RANKERN, build-up data are held as cubic polynomials[2] such that

$$B(\tau) = \sum_{i=0}^{3} A_i \tau^i ,$$

the data varying with material and energy and only one set of build-up data being used for a given configuration.

Such "point isotropic" build-up data are suited to situations such as path A in Figure 1, when the dominant penetration path is normal to the shield geometry.

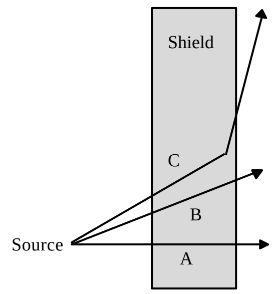

Figure 1 - Normal and Oblique Build-up

As this penetration path becomes oblique to the shield, see path B, the build-up factor is corrected[3] by RANKERN such that

$$B(\theta_o, \tau_o) = B(0, \tau_o) e^{-(\sec(\beta\theta_o)-1)\tau_o}$$

where $B(\theta_o)$ is the oblique build-up factor, B(0) is the build-up factor for normal incidence, β is an empirically derived angle coefficient typically $0.80<\beta<0.95$, and the variables θ_o and τ_o refer to the incident obliquity and shield thickness in mfp respectively.

Depending on the thickness of the shield, this technique works successfully up to angles of obliquity of about 25°, after which it becomes increasingly inaccurate. As a rule-of-thumb, if the result using oblique build-up is more than a factor ten higher than that using normal build-up, the result is unreliable. Ultimately, see path C, the oblique build-up treatment breaks down and intermediate scatter events have to be treated explicitly.

III. Scatter Calculations

Path C in Figure 1 can be analysed by considering two legs of the path connected by a single scatter event. Then, the flux at the dose-point due to radiation that undergoes scattering within volume V_s may be written:

$$\phi = \int_{V_s}\int_{V} \frac{SBe^{-\tau_o}}{4\pi r_o^2} \frac{\sigma_s e^{-\tau_s}}{4\pi r_s^2} dvds \ ,$$

where r_o is the penetration distance between the source and scatter point, r_s is the penetration distance between the scatter and dose-points, σ_s is the differential scattering cross-section, and τ_o and τ_s are the total penetration distances in mfp along r_o and r_s respectively. The method can be extended to higher orders of scatter, and is also applicable to skyshine studies[1].

With this algorithm a number of factors need to be considered over a conventional point-kernel calculation, namely the scattering cross-section, the energy of the gamma-ray after scatter, the build-up data to be applied, and the size of the scatter volume.

1. Scatter cross-section

The scattering cross-sections for the shield materials are based upon the Klein-Nishina equation and the magnitudes of the cross-sections are derived using electron density values for the materials. The code accesses such data automatically by knowing in which material the scatter occurs.

2. Energy after scatter

The energy E of the scattered gamma-ray is given by the Compton expression such that

$$E = E_o \left(1 + \frac{E_o}{0.511} \left(1 - \cos\theta_s\right) \right)^{-1} ,$$

where E_o is the source energy and θ_s is the angle of scatter.

The value of E determines the transmission cross-section and build-up data (see below) to be used for the second leg of the route from source to dose-point.

3. Build-up data

Since one scatter has been treated explicitly, it may be thought that build-up need only be applied to one leg. However, as presented later, experience has shown that this leads to underestimation of the result. Even though the approach is physically pessimistic, it is therefore recommended that build-up is applied to both legs of the path, and this is taken as the default treatment by the code.

4. Size of scatter volume

The region of the shield where scatter is to occur must be large enough for all the important events to be included. This is dictated by the dominant penetration paths through the system, which may be complicated; but in general it leads to scatter volumes approximately 3mfp thick in the regions of the shield bordering the void or duct. These are specified in RANKERN as simple bodies, e.g. boxes or cylinders, which may usefully be subdivided into smaller units. The reason for subdivision is that since RANKERN determines the contribution to the result from each subdivision and alters its sampling probability as the calculation progresses, the more important subdivisions are sampled more often and those giving a negligible contribution are effectively neglected. It is therefore advisable to make the scatter volume larger than necessary, in the knowledge that the code will optimise the calculation itself.

IV. Reflection

In some situations, e.g. back-scatter from a wall, the scatter can be represented by reflection events distributed over an area A. In this case, the scatter cross-section is replaced by the probability of reflection, or albedo β thus

$$\phi = \int_{A}\int_{V} \frac{SBe^{-\tau_o}}{4\pi r_o^2} \frac{\beta e^{-\tau_s}}{4\pi r_s^2} dvda$$

Similar considerations to scatter calculations apply regarding the use of build-up data and the size and specification of the reflection surface. Again, the energy of the reflected gamma-ray is given by the Compton expression, but albedo data have to be defined.

1. Albedo data

The value of β is derived from the so-called "current dose albedo"[4]

$$\alpha_D = \frac{CK(\theta_s)10^{26} + C'}{1 + \cos\theta_o \sec\theta},$$

where C and C' are fitted parameters corresponding to single scatter and multiple scatter components respectively; $K(\theta_s)$ is the Klein-Nishina energy scattering coefficient for scattering angle θ_s, and θ_o and θ are the incident and emergent polar angles respectively as measured from the normal to the surface.

Values of C and C' have been obtained by fitting Monte Carlo calculations for broad parallel beams of photons incident at various angles to the surface of the media[5], and are stored within RANKERN for various materials and ranges of energy.

The current dose albedo is converted to β by scaling by the dose-rate conversion factors before and after reflection, i.e.

$$\beta = \alpha_D \frac{D(E_o)}{D(E)},$$

where D(E) is the dose conversion factor for energy E.

V. Scatter or Reflection?

It is vital that the dominant penetration paths through the system being analysed are identified, and usually separate RANKERN calculations are performed for each path in order to assess their relative contributions. Having identified the paths and determined where intermediate events are to occur, a decision must be made whether scatter or reflection is to be used. In some circumstances, like Figure 1, scatter is the obvious choice since reflection in inappropriate. In other cases, however, the choice is not so straightforward.

Consider the system illustrated in Figure 2, where source and detector are separated by an effectively black body. The route from source to detector therefore only involves interactions in the wall.

With the wall modelled as a reflection surface, all the important paths from source to detector are included in the calculation.

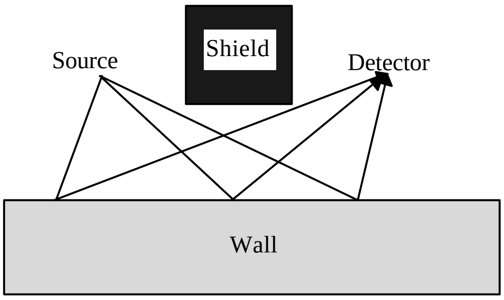

Figure 2 - Reflection

However, if the shield is nearer the wall, as shown in Figure 3, this is not the case since some of the paths are cut off. In such a situation, scatter within the wall has to be included, as shown by the dashed lines.

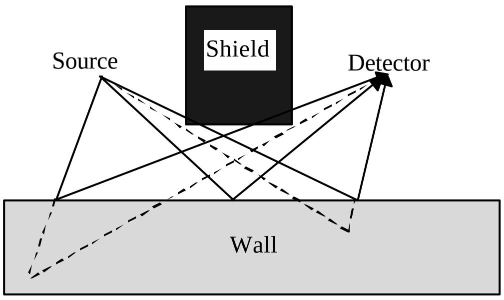

Figure 3 - Single Scatter

When the shield is right up against the wall, as illustrated in Figure 4, single scatter events are also cut off and multiple scatter is required as shown by the dotted line.

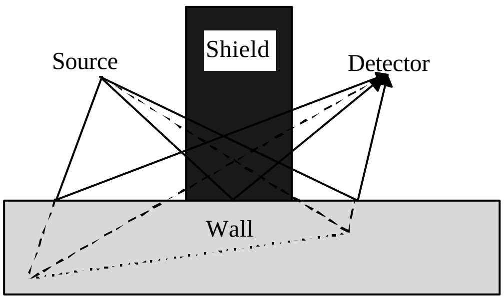

Figure 4 - Multiple Scatter

It is therefore very important that the analyst identifies the important scatter/reflection paths and uses the appropriate order and size of scatter/reflection region.

VI. Validation

The accuracy of RANKERN calculations that use scatter and/or reflection has been considered for a number of different scenarios[6]. Experiments in the ARCAS facility at AEA Technology's Winfrith site were performed, which consisted of Co^{60} sources, dosimeters, and shield configurations built from aluminium, iron or lead.

1. Oblique penetration

A study of oblique penetration through iron slabs, schematically similar to Figure 1, indicated that RANKERN predictions of dose-rate were about 20% pessimistic compared with measurement until the oblique build-up method began to break down. This was the basis of the rule-of-thumb mentioned in section II.

2. Single scatter or reflection

An experimental configuration was built which resembled Figure 2, with an aluminium wall and a lead shield. With a single event, either scatter or reflection, RANKERN predicted dose-rates within 20% of measurement. Since treating reflection is quicker than scatter, because of numerical integration over an area rather than a volume, the former is recommended for such situations.

Similarly, a configuration resembling Figure 3 was constructed. As expected, a single reflection led to underestimation of the result, so scatter should be used instead.

3. Multiple scatter

The accuracy of a multiple scatter calculation was assessed by a configuration similar to Figure 4. In this case, RANKERN's predictions of dose-rate were between a factor 1.5 and 4 pessimistic, this being mainly due to the conservative approach of applying build-up to all three legs. It is worth noting that applying build-up to only the last leg led to underpredictions of a factor five, hence the recommendation to apply build-up to all legs of the calculation.

In practice multi-order calculations become increasing longer to execute, and for orders higher than two it is usual to resort to the more rigorous Monte Carlo code MCBEND[7]. Since the two codes use the same geometry package, transferring from one calculational method to the other is relatively straightforward.

4. Penetration following reflection

This assessment considered penetration of an aluminium slab following an initial reflection event, as illustrated in Figure 5.

In this case, RANKERN's predictions of dose-rate were up to a factor two pessimistic compared with that measured. This is associated with the energy of the reflected gamma-ray, which, as noted in section IV, is given by the Compton expression. Hence, although the value of albedo takes multiple scatter into account, the reflected gamma-ray spectrum does not. This leads to the reflected spectrum being too hard, which in turn leads to too little material attenuation and an over-prediction of the result. Developments are in hand to improve RANKERN's treatment of reflection in this respect.

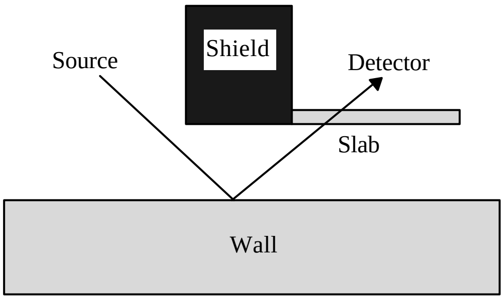

Figure 5 - Penetration following Reflection

5. Multi-legged ducts

A common problem in shield design arises from the need to provide penetrations for services such as ventilation, cooling, sampling, chemical dosing and instrumentation into cells containing intense sources of gamma-rays arising from fission products. The pipes that serve these functions must be bent to avoid direct paths through the shield, which would lead to excessive external dose-rates. Often, the pipes have two bends within the shield, thus dividing them into three legs, the first and third legs being normal to the faces of the shield. The attenuation of gamma-rays along a bent pipe of given diameter in a shield of specified thickness is determined by the offset between the first and third legs and the angle between the second leg and the other two.

The optimisation of pipe configurations requires the calculation of the dose-rate at the cold end of the pipe due to gamma-rays which have streamed along all three legs of the pipe. This final analysis is an example of the application of RANKERN to this typical streaming problem, and such a three-legged duct is illustrated in Figure 6.

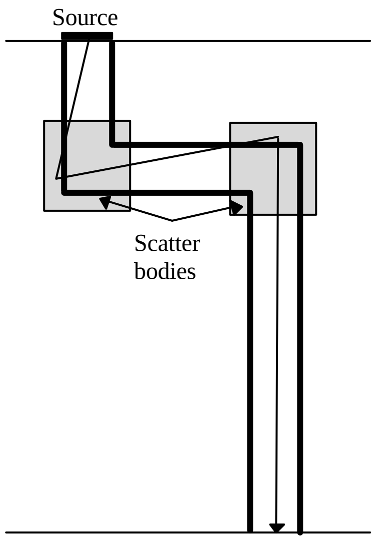

Figure 6 - Multi-legged Duct

The duct is cylindrical, of diameter 150mm and is lined with 6mm of steel. The three legs are 440mm, 600mm and 1375mm long respectively. The source is a disc with source energy of 2.25MeV, and the dose-rate at the end of the third leg of the duct is required. First and second order scatter bodies were positioned at the first and second corners respectively as seen from the source. This defines the route from source to dose-point. At the end of the duct, the dose-rate predicted by RANKERN was a factor 2.4 higher that predicted using the MCBEND Monte Carlo code. This is consistent with the accuracy of multiple scatter calculations described in section VI.3.

As part of such an analysis, the importance of other routes through the system would be investigated separately, e.g. those shown in Figure 7 which show first and third order penetration routes. For some of the events, e.g. scatter along the third leg of the duct, reflection could be specified instead.

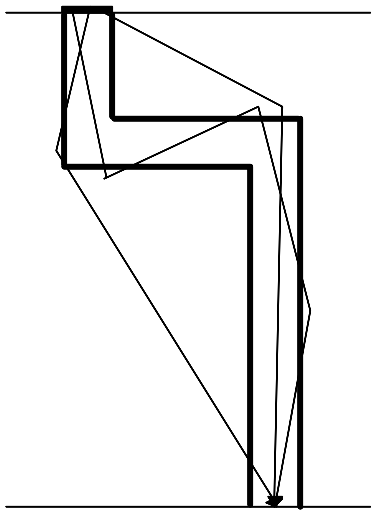

Figure 7 - Routes through the Duct

Whether the various routes provide significant contributions to the result depends on the configuration of the duct, and as part of its output RANKERN indicates the relative importance of each route and order of scatter. In this way a thorough understanding of the way radiation penetrates the shield is acquired.

VII. Visualisation

RANKERN is accompanied by a number of graphics packages[8] for visualising the geometry in 2 (VISAGE) or 3 (VISTA-RAY) dimensions. Additionally, VISTA-WIRE, provides wire-frame drawings of the configuration - including source bodies and scatter/reflection regions as illustrated in Figure 8. This enables easy checking of the positioning of the geometry, source and scatter/reflection bodies.

VIII. Summary

RANKERN provides a rapid design method for gamma-ray shielding, featuring the treatment of intermediate scatter and reflection events between source and dose-point which allow streaming calculations to be performed. The accuracy of such calculations has been assessed against measurements and more rigorous Monte Carlo calculations, with RANKERN generally producing safely pessimistic results.

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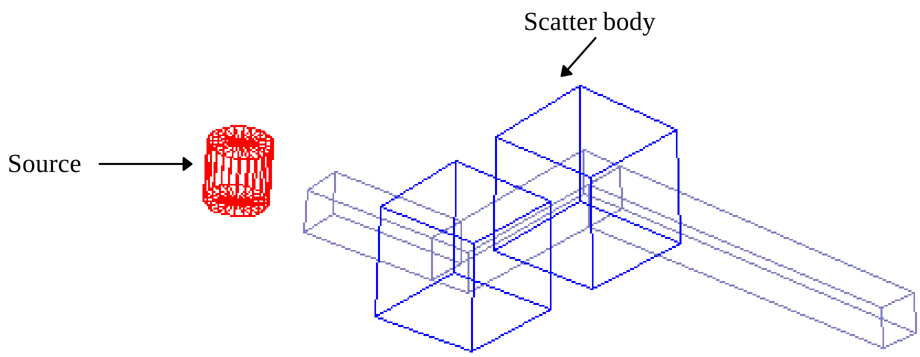

Figure 8 - VISTA-WIRE Representation of a Multi-legged Duct